

RAPPELS : $\lim_{x \rightarrow -\infty} x^n e^x = 0$

$$\lim_{x \rightarrow +\infty} \frac{e^x}{x^n} = +\infty$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

EXERCICE 3B.1 Déterminer les limites suivantes :

a. $\lim_{x \rightarrow +\infty} \frac{e^{x+5}}{2x+3} =$

b. $\lim_{x \rightarrow +\infty} \frac{e^{1+x^2}}{4x^5 - 7} =$

c. $\lim_{x \rightarrow -\infty} (4x-3)e^{5x-7} =$

d. $\lim_{x \rightarrow -\infty} (2-5x^4)e^{3x^2} =$

e. $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{8x} =$

f. $\lim_{x \rightarrow 0} \frac{e^{-x^3} - 1}{9x} =$

g. $\lim_{x \rightarrow 0} \frac{e^{4x^2} - 1}{x^5} =$

CORRIGE – Notre Dame de La Merci – Montpellier**EXERCICE 3B.1** Déterminer les limites suivantes :

$$\text{a. } \lim_{x \rightarrow +\infty} \frac{e^{x+5}}{2x+3} = \lim_{x \rightarrow +\infty} \frac{e^{x+5}}{x+5} \times \frac{x+5}{2x+3}$$

$$\text{on pose : } X = x+5 \quad \text{AINSI : } \lim_{x \rightarrow +\infty} \frac{e^{x+5}}{x+5} = \lim_{X \rightarrow +\infty} \frac{e^X}{X} = +\infty$$

$$\lim_{x \rightarrow +\infty} \frac{x+5}{2x+3} = \lim_{x \rightarrow +\infty} \frac{x \left(1 + \frac{5}{x}\right)}{x \left(2 + \frac{3}{x}\right)} = \lim_{x \rightarrow +\infty} \frac{1 + \frac{5}{x}}{2 + \frac{3}{x}} = \frac{1}{2}$$

$$\text{Par produit : } \lim_{x \rightarrow +\infty} \frac{e^{x+5}}{2x+3} = +\infty$$

$$\text{b. } \lim_{x \rightarrow +\infty} \frac{e^{1+x^2}}{4x^5-7} = \lim_{x \rightarrow +\infty} \frac{e^{1+x^2}}{1+x^2} \times \frac{1+x^2}{4x^5-7} = \lim_{x \rightarrow +\infty} \frac{e^{1+x^2}}{1+x^2} \times \frac{x^2 \left(\frac{1}{x^2} + 1\right)}{x^5 \left(4 - \frac{7}{x^5}\right)} = \lim_{x \rightarrow +\infty} \frac{e^{1+x^2}}{1+x^2} \times \frac{\frac{1}{x^2} + 1}{x^3 \left(4 - \frac{7}{x^5}\right)}$$

$$\text{on pose : } X = 1+x^2 \quad \text{AINSI : } \lim_{x \rightarrow +\infty} \frac{e^{1+x^2}}{1+x^2} = \lim_{X \rightarrow +\infty} \frac{e^X}{X} = +\infty$$

$$\lim_{x \rightarrow +\infty} \frac{\frac{1}{x^2} + 1}{x^3 \left(4 - \frac{7}{x^5}\right)} = 0^+$$

$$\text{Par produit : } \lim_{x \rightarrow +\infty} \frac{e^{1+x^2}}{4x^5-7} = +\infty$$

$$\text{c. } \lim_{x \rightarrow -\infty} (4x-3)e^{5x-7} = \lim_{x \rightarrow -\infty} \frac{4x-3}{5x-7} \times (5x-7)e^{5x-7} = \lim_{x \rightarrow -\infty} \frac{x \left(4 - \frac{3}{x}\right)}{x \left(5 - \frac{7}{x}\right)} \times (5x-7)e^{5x-7}$$

$$= \lim_{x \rightarrow -\infty} \frac{4 - \frac{3}{x}}{5 - \frac{7}{x}} \times (5x-7)e^{5x-7}$$

$$\text{on pose : } X = 5x-7 \quad \text{AINSI : } \lim_{x \rightarrow -\infty} (5x-7)e^{5x-7} = \lim_{X \rightarrow -\infty} X e^X = 0^-$$

$$\lim_{x \rightarrow -\infty} \frac{4 - \frac{3}{x}}{5 - \frac{7}{x}} = \frac{4}{5}$$

$$\text{Par produit : } \lim_{x \rightarrow -\infty} (4x-3)e^{5x-7} = 0$$

$$\begin{aligned}
 \text{d. } \lim_{x \rightarrow -\infty} (2-5x^4)e^{3x^3} &= \lim_{x \rightarrow -\infty} \left(\frac{2-5x^4}{3x^3} \right) \times (3x^3)e^{3x^3} = \lim_{x \rightarrow -\infty} \left(\frac{x^4 \left(\frac{2}{x^4} - 5 \right)}{3x^3} \right) \times (3x^3)e^{3x^3} \\
 &= \lim_{x \rightarrow -\infty} \left(\frac{x \left(\frac{2}{x^3} - 5 \right)}{3} \right) \times (3x^3)e^{3x^3} = \lim_{x \rightarrow -\infty} \frac{\frac{2}{x^3} - 5}{3} \times x \times (3x^3)e^{3x^3} \\
 &= \lim_{x \rightarrow -\infty} \frac{\frac{2}{x^3} - 5}{3} \times \frac{3x^3}{3x^2} \times (3x^3)e^{3x^3} = \lim_{x \rightarrow -\infty} \frac{\frac{2}{x^3} - 5}{3} \times \frac{1}{3x^2} \times (3x^3)^2 e^{3x^3}
 \end{aligned}$$

on pose : $X = 3x^2$ AINSI : $\lim_{x \rightarrow -\infty} (3x^3)^2 e^{3x^3} = \lim_{X \rightarrow -\infty} X^2 e^X = 0^+$

$$\lim_{x \rightarrow -\infty} \frac{\frac{2}{x^3} - 5}{3} \times \frac{1}{3x^2} + \lim_{x \rightarrow -\infty} \frac{\frac{2}{x^3} - 5}{9x^2} = 0^+$$

Par produit : $\lim_{x \rightarrow -\infty} (2-5x^4)e^{3x^3} = 0$

$$\text{e. } \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{8x} = \lim_{x \rightarrow 0} \frac{2x}{8x} \times \frac{e^{2x} - 1}{2x} = \lim_{x \rightarrow 0} \frac{1}{4} \times \frac{e^{2x} - 1}{2x}$$

on pose : $X = 2x$ AINSI : $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{2x} = \lim_{X \rightarrow 0} \frac{e^X - 1}{X} = 1$

Par produit : $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{8x} = \frac{1}{4}$

$$\text{f. } \lim_{x \rightarrow 0} \frac{e^{-x^3} - 1}{9x} = \lim_{x \rightarrow 0} \frac{-x^3}{9x} \times \frac{e^{-x^3} - 1}{-x^3} = \lim_{x \rightarrow 0} \frac{-x^2}{9} \times \frac{e^{-x^3} - 1}{-x^3}$$

on pose : $X = -x^3$ AINSI : $\lim_{x \rightarrow 0} \frac{e^{-x^3} - 1}{-x^3} = \lim_{X \rightarrow 0} \frac{e^X - 1}{X} = 1$

$$\lim_{x \rightarrow 0} \frac{-x^2}{9} = 0^-$$

Par produit : $\lim_{x \rightarrow 0} \frac{e^{-x^3} - 1}{9x} = 0$

$$\text{g. } \lim_{x \rightarrow 0} \frac{e^{4x^2} - 1}{x^5} = \lim_{x \rightarrow 0} \frac{4x^2}{x^5} \times \frac{e^{4x^2} - 1}{4x^2} = \lim_{x \rightarrow 0} \frac{4}{x^3} \times \frac{e^{4x^2} - 1}{4x^2}$$

on pose : $X = 4x^2$ AINSI : $\lim_{x \rightarrow 0} \frac{e^{4x^2} - 1}{4x^2} = \lim_{X \rightarrow 0} \frac{e^X - 1}{X} = 1$

$$\lim_{x \rightarrow 0^+} \frac{4}{x^3} = +\infty \quad \text{et} \quad \lim_{x \rightarrow 0^-} \frac{4}{x^3} = -\infty$$

Par produit : $\lim_{x \rightarrow 0^+} \frac{e^{4x^2} - 1}{x^5} = +\infty$ et $\lim_{x \rightarrow 0^-} \frac{e^{4x^2} - 1}{x^5} = -\infty$